ROBUST EVALUATION MATRIX
TOWARDS A MORE PRINCIPLED OFFLINE EXPLORATION OF INSTRUCTIONAL POLICIES

Shayan Doroudi, Vincent Aleven, Emma Brunskill
April 20, 2017
\[
\frac{7}{8} \div \frac{2}{7}
\]

\[
\frac{2}{4} \times \frac{2}{7}
\]

\[
\frac{3}{4} + \frac{1}{8}
\]

\[
\frac{1}{4} < \frac{1}{3}?
\]

\[
\frac{2}{4} \times \frac{2}{7}
\]

\[
\frac{3}{4} + \frac{1}{8}
\]

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\frac{2}{4} \times \frac{2}{7}
\]
• Experiments take time and resources.
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• Could be uninformative.
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• Could be uninformative.
• Large space of possible interventions. How do we choose what to try?
Can we tell how well an intervention will do without running it?
How can we determine what intervention is best?
• Can use simulated students!
• Can use simulated students!
• Fit student model to past data.
- Can use simulated students!
- Fit student model to past data.
- Use student model to simulate an intervention.
Past Data

\[
\frac{3}{4} + \frac{1}{8}
\]

\[
\frac{1}{4} < \frac{1}{3}?
\]

\[
\frac{2}{4} \times \frac{2}{7}
\]

\[
\frac{7}{8} \div \frac{2}{7}
\]

Intervention

Intro to Calculus
Past Data

\[ \frac{3}{4} + \frac{1}{8} \]

\[ \frac{1}{4} < \frac{1}{3}? \]

\[ \frac{2}{4} \times \frac{2}{7} \]

\[ \frac{7}{8} \div \frac{2}{7} \]

Intervention

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\[ \frac{2}{4} \times \frac{2}{7} \]

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Past Data

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\[ \frac{2}{4} \times \frac{2}{7} \]

\[ \frac{7}{8} \div \frac{2}{7} \]

Intervention

**Instructional Policy:**
Method of sequencing activities, possibly adaptive with respect to student state.
But what if the student model is inaccurate?
Simulate instructional policies on *several* student models!
Simulate instructional policies on \textit{several} student models! Use many models we expect to be wrong, rather than using one model we hope to be right.
Student Model

Instructional Policy
\[ \frac{1}{4} < \frac{1}{3} ? \]

Student Model

Instructional Policy

\[ \frac{1}{4} < \frac{1}{3} ? \]
\[
\frac{1}{4} < \frac{1}{3}?
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\[ \frac{3}{4} + \frac{1}{8} \]

\[ \frac{2}{4} \times \frac{2}{7} \]
\[
\frac{1}{4} < \frac{1}{3} ? \quad \frac{3}{4} + \frac{1}{8} \quad \frac{2}{4} \times \frac{2}{7}
\]

80% on Posttest

Student Model

Instructional Policy
# Robust Evaluation Matrix (REM)

<table>
<thead>
<tr>
<th>Student Model 1</th>
<th>Policy 1</th>
<th>Policy 2</th>
<th>Policy 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{SM_1,IP_1}$</td>
<td>$V_{SM_1,IP_2}$</td>
<td>$V_{SM_1,IP_3}$</td>
<td></td>
</tr>
<tr>
<td>$V_{SM_2,IP_1}$</td>
<td>$V_{SM_2,IP_2}$</td>
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</tbody>
</table>
CASE STUDY: FRACTIONS TUTOR

1. Compare these fractions using the cross-multiplication strategy.

   \[
   \frac{\cancel{4} \times \cancel{5}}{\cancel{5}} > \frac{\cancel{9} \times \cancel{10}}{\cancel{10}}
   \]

2. Finally, reduce the sum to lowest terms:

   \[
   \frac{2}{10} + \frac{3}{4} = \frac{19}{20} = \frac{19}{20}
   \]
What goes wrong if we simulate on only a single student model?
CASE STUDY: SINGLE MODEL SIMULATION

- Used prior data to fit G-SCOPE Model (Hallak et al., 2015).
CASE STUDY: SINGLE MODEL SIMULATION

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- Used G-SCOPE Model to derive new Adaptive Policy.
CASE STUDY: SINGLE MODEL SIMULATION

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- Wanted to compare Adaptive Policy to a Baseline Policy (fixed, spiraling curriculum).
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• Used G-SCOPE Model to derive new Adaptive Policy.
• Wanted to compare Adaptive Policy to a Baseline Policy (fixed, spiraling curriculum).
• Simulated both policies on G-SCOPE Model to predict posttest scores (out of 16 points).
## CASE STUDY: SINGLE MODEL SIMULATION

<table>
<thead>
<tr>
<th>Simulated Results</th>
<th>Baseline</th>
<th>Adaptive Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.9 ± 0.9</td>
<td>9.1 ± 0.8</td>
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## Case Study: Single Model Simulation

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<tr>
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<th>Baseline</th>
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<tbody>
<tr>
<td>Simulated Results</td>
<td>$5.9 \pm 0.9$</td>
<td>$9.1 \pm 0.8$</td>
</tr>
<tr>
<td>Experimental Results</td>
<td>$5.5 \pm 2.6$</td>
<td>$4.9 \pm 1.8$</td>
</tr>
</tbody>
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• Used by Chi et al. (2011) and Rowe et al. (2014) in educational settings.
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• But in experiment, no significant difference found (Rowe and Lester, 2015).
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• But in experiment, no significant difference found (Rowe and Lester, 2015).
• If each policy is only simulated on the student model that was used to derive it, a sub-optimal policy might be predicted to be better than the optimal policy under the true student model (Mandel et al., 2014).
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• If each policy is only simulated on the student model that was used to derive it, a sub-optimal policy might be predicted to be better than the optimal policy under the true student model (Mandel et al., 2014).
  • Even with an infinite amount of data!
## Case Study: Robust Evaluation Matrix

<table>
<thead>
<tr>
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<th>Adaptive Policy</th>
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</thead>
<tbody>
<tr>
<td>G-SCOPE Model</td>
<td>5.9 ± 0.9</td>
<td><strong>9.1 ± 0.8</strong></td>
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### Bayesian Knowledge Tracing Model

### Deep Knowledge Tracing Model
CASE STUDY: ROBUST EVALUATION MATRIX

<table>
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<th>Model</th>
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<tr>
<td>G-SCOPE Model</td>
<td>5.9 ± 0.9</td>
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<td>6.5 ± 0.8</td>
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**CASE STUDY: ROBUST EVALUATION MATRIX**

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## Case Study: Robust Evaluation Matrix

<table>
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<tbody>
<tr>
<td>G-SCOPE Model</td>
<td>5.9 ± 0.9</td>
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See paper for another retrospective analysis of prior work (Rafferty et al., 2015) showing REM can be used:
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- to find good policies, robust to the choice of the model
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• to find good policies, robust to the choice of the model
• to spot bad policies that single model simulation could not catch
• Simulating instructional policies on various student models can help determine what policy is best (if any) in advance of running an experiment.
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• Showed retrospective analysis of how REM could have been used to inform experiment.
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• Showed retrospective analysis of how REM could have been used to inform experiment.

• Next Step: Close the loop.
Use many models we expect to be wrong, rather than using one model we hope to be right.

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  • 20 sequential decisions $\Rightarrow$ need over $2^{20}$ students
IMPORTANCE SAMPLING

- Estimator that gives unbiased and consistent estimates for a policy!
- Can have very high variance when policy is different from prior data.
- Example: Worked example or problem-solving?
  - 20 sequential decisions $\Rightarrow$ need over $2^{20}$ students
  - 50 sequential decisions $\Rightarrow$ need over $2^{50}$ students!